Approximation Taxonomy of Metric TSP

(Update: August 2013)

The list below presents the best up to now known upper and lower approximation bounds for the instances of metric TSP (we refer also to another source on approximation algorithms for metric TSP [8]). It is intended to codify (at one glance) the (many) recent developments and improvements on the approximability of that problem. It is hoped to be useful in further research on the possible improvements of the underlying approximation bounds.

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1 General Metric TSP [*]

- $\frac{3}{2}$-approximation [6]
- hard to approximate within a factor less then $\frac{123}{122}$ [9]

2 Graphic TSP

- $\frac{7}{5}$-approximation [14]
- hard to approximate within a factor less then $\frac{535}{534}$ [11]

3 (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a factor less then $\frac{535}{534}$ [11]

4 Dense (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a constant $\frac{535}{534}$ [11] [7]

5 Cubic Graphic TSP

- $\frac{4}{3}$-approximation [5], [13]
- hard to approximate within a factor less then $\frac{1153}{1152}$ [10]

[*]The underlying annotations to the problems can be found in the enclosed references-update.

6 Subcubic Graphic TSP

- $\frac{4}{3}$-approximation [13]
- hard to approximate within a constant $\frac{685}{684}$ [10]

7 Cubic (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a factor less then $\frac{1141}{1140}$ [10]

8 Subcubic (1,2)-TSP

- $\frac{8}{7}$-approximation [3]
- hard to approximate within a constant $\frac{673}{672}$ [10]

9 Asymmetric TSP

- $O(\frac{log\ n}{log\ log\ n})$-approximation [2]
- hard to approximate within a factor less then $\frac{75}{74}$ [9]

10 Asymmetric (1,2)-TSP

- $\frac{5}{4}$-approximation [4]
- hard to approximate within a factor less than $\frac{207}{206}$ [11]

11 Geometric TSP (arbitrary l_p metric)

- PTAS for every fixed dimension [1], [12]
- APX-hard for *log n* dimensions [15]

. $\square$

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